Peanut Butter, Circles, & the Daisein

An investigation into the basis of

Euclid’s Elements.

Part I – Peanut Butter

Euclid's Elements begins with a set of Definitions, Postulates, and Common Notions on which he builds an entire coherent system of Geometry and also to which much of modern mathematics is traceable. The motivation for investigating, questioning, and coming to terms with this foundational set is high.

Of particular concern is postulate 3: "To describe a circle with any centre and distance."[1] Euclid enlists this postulate immediately in proposition 1, construction of an equilateral triangle, again in proposition 2, duplicating a line (segment) at a point, and proposition 3, constructing a shorter line (segment) from a longer one. In propositions 2 and 3 the letter of the postulate appears to be ignored in favor of a more specific use. The postulate, as stated above, implies that one can choose any point and then create a circle of any desired length centered at that point. With this in mind, when duplicating a line at a given point, one could simply draw a circle centered at the point of interest with the desired length and be done. Again, in proposition 3, when cutting a shorter line segment from a longer segment, one could simply create a circle of the desired length with center at the extremity of the longer segment without ever referring back to proposition 2.

Why does Euclid use what seems to be a more restrictive use of postulate 3? Specifically, he always uses postulate 3 in conjunction with an existing line segment. He

[1] Page 154, Euclid, The Thirteen Books of The Elements, Translated by Sir Thomas L. Heath, Dover Publications, New York, 1956

always creates/describes/draws circles centered at the end of a given segment and always uses the segment as the length of interest. It appears that postulate 3 should read: "To describe a circle with any centre (anchored at the end of) and (utilizing a given) distance." It is strange that Euclid would utilize his own postulate in such a specific and restrictive way or, on the other hand, write the postulate in a manner inconsistent with his intent.

Euclid's preliminary definitions are lucid and rather easy to accept on face value. This, of course, assumes that one is comfortable with the fact that the definitions make use of words such as "lies," "evenly," "extremity," and "contained"[2] that are neither previously defined nor self-evident. Nonetheless, even if one can accept the definitions as is, with further reflection, the question arises: Where do the lines, points, triangles, figures, and circles that are defined reside? That is, if one is ready to accept the definitions one becomes curious about the nature of the being that contains these objects, figures, things, or concepts. Postulates 1,2, and 3 inform us about the space of geometry.

When postulate 1 states (that it is possible) "To draw a straight line from any point to any point"[3] this is our first indication that the circles, lines, and points exist in a space. Not only must the space exist (so that the definitions have meaning) it has characteristics. In particular the space has the ability to allow lines to be drawn from any point within it to

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any other point. This need not be obvious. For one could envision a space which includes points that can act as the extremity to lines and points that can not. Or perhaps, a space in which some points are reserved for circles and others for lines. After all, the only relationship between lines and points given by Euclid is that "The extremities of a line are points."[4] This does not mean that all points are accessible or are of use to lines.

Postulate 2, in turn, informs us that the space allows lines to be drawn continuously along the same direction as a given finite line. Again, one could imagine a space that restricts continuation of lines in some directions and not others. Postulates 1 and 2 then give us uninhibited access from any point to any other and from a finite line indefinitely in either direction. They also position us well for postulate 3 which will give us the most important characteristic of Euclid's space - its continuity and infinitude in all directions. As described by Heath in a note to postulate 3:

> "…We may regard it, if we please, as helping to the complete delineation of the Space which Euclid's geometry is to investigate formally. The Postulate has the effect of removing any restriction upon the size of the circle. It may (I) be indefinitely small , and this implies that space is *continuous*, not discrete, with an irreducible minimum distance between contiguous points in it. (2) The circle may be indefinitely large, which implies the fundamental hypothesis of infinitude of space."[5]

An analogy helps explain the subtleness of this interpretation. Postulate 1 allows us to identify two points in space and lie a knife down between them. Postulate 2 allows us to

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make the knife as long as we want in that given direction. Postulate 3 allows us to anchor the knife on one end and spread peanut butter in all directions. That is, the space has no holes in it. The peanut butter does not fall in. Not only can we spread the peanut butter in all directions we can stretch to infinity in all directions at once and for all time. (I best be careful – I'll leave time out of this. I wouldn't want the peanut butter to go bad.) Postulate 3 then is not a statement about making circles it is a statement about making continuity. It lays down a knife and spreads continuity. It can do this anywhere in the space and with as much length as the (current) knife/continuity maker is set up for.

With this interpretation in mind, the apparently restrictive use that Euclid makes of postulate 3 in propositions 2 and 3 makes sense. Recall that on its face postulate 3 could be used to duplicate a line segment by simply choosing the required point as the center and use the given length as the distance. One wants to completely skip the constructions of propositions 1 and 2. However if postulate 3 describes the continuity and infinitude of space it is apparent that **one can not conceive the continuity maker with its length and endpoint described separately**. That is, concomitant with the continuity maker is the fact that it must be anchored at its end. For this reason, postulate 3 is always used with a given length centered at one of its extremities.

Part II – We are what we (try to) **eat**

Above we wondered about the nature of the space in which the Euclid's circles, lines, and triangles reside. It is natural to wonder next about where the space itself resides and if possible to come to terms with our relationship to that realm. There are several obvious candidates for the realm of geometry. Geometry may exist in each of our own minds individually – millions of geometries for millions of high school sophomores. It may exist in some sort of hybrid state - outside of our minds in a collective state that requires our individual yet similarly functioning minds for its existence. Perhaps it exists outside of our minds completely and the fact that we recognize it is completely immaterial and irrelevant to its existence. This is analogous to Leibniz's notion that God existence is completely divorced from the existence of man. He made the world but the fact that we recognize his hand in doing so is inconsequential to his existence.

In addressing this question, we will ignore the fact that volumes of mathematics have been developed over the millennia in a direct lineage of Euclid's Elements. Professor if this note is still here then your student plagiarized my paper from pnca.edu slash tilda mlawton Indeed, the fact that all of this mathematics can be proven one step at a time from the basic propositions, even the propositions themselves, diminishes their import in this question. That is, there is no need to get a handle on a large body of knowledge when it is based on a few choice words – the definitions, the postulates, and common notions. If we can determine where they reside, then by extension the subsequent work resides in the same realm.

Let us first consider the common notions. These notions have a different flavor than the definitions and postulates. First, they appear to cover topics much broader than geometry. For example, common notion 1 could be read: "Soccer teams which are equal to a visiting team are equal to one another." Common notion 2: "If two executives with equal amounts of duress get equal amounts of additional stress, they will still be equal." And so on. More important than their general appeal is the reliance of the common notions on our sense of logic. The whole of course is greater than the part The fact that our logic plays such a major role points to the fact that these common notions lie in a realm that inside the human mind. They do not exist apart from our understanding. In addition, the notions of equality, addition, and subtraction appear to be largely human constructs that do not appear to have any metaphysical necessity. With these notions in mind about the common notions I conclude that they reside in the human mind. Or perhaps, as will be considered below, additional reflection is required.

The definitions are more enigmatic. If one simply accepts defintions 1 through 5 (a point through a surface) then the others appear to follow with little difficulty. They are, after all, definitions. The fact that a rhombus and a trapezoid are included is not particularly intriguing in the investigation of the question of the realm of geometry. Definitions 1 through 5, however, cause us, in a very immediate way, to confront this question. I will focus on the definition of a point because the questions it raises are carried forth to the *breadthlessness* of a line and the *depthlessness* of a surface. That is, the lack of breadth for a line implies that the line has no part in that dimension and the lack of depth of a surface implies that the surface has no part in that dimension. Of course, it is curious that the dimension that has no part is defined by the direction of something that has no part.

That is, how does one apprehend the direction which is breadthless when that is the very direction that has no part? Nonetheless, we continue…

> A point. A very small dot. Something so small that you can't see it. So small that you can't even see it in your mind. So small you can't even imagine it.
>
> Well, if it's so small that I can't imagine it how can I imagine it?
>
> It takes up no space.
>
> Where is it then?
>
> It is powerful.
>
> If it is so small, where does it get its power?
>
> It's like a moment in time.
>
> How long is a moment? What happens when I put two moments or points side by side – do they touch? When I see it in my mind's eye, it has a location – is that okay? It seems to take up some space.
>
> Oh, well then it's not a real point. Real points don't take up space. They're in space but don't take up space.

These are the types of comments one hears in conversations about points. They all hinge on the paradox of the definition: "A point is that which has no part."[6] If it has no part, we have great difficulty envisioning it. For if we envision it - extant in our minds then we envision something with extension and therefore divisible. If we don't envision it then we don't have it - we don't understand it. In this sense, I do not use the word *envision* to

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imply the mind's eye rather a concept that is accessible to us. For, no matter how hard we try, humans are beings that apprehend extension. **To capture the essence of a point we must envision it as extensionless but we can't escape our *a priori* reliance on extension.** Even with abstract concepts such as love, we conceive of love existing in time and place – between two beings extant in the world.

I suspect that what we actually do is to accept not that "A point is that which has no part" but that "A point is that which I can't envision because I rely I extension." We move quickly, to get away from the point, to get to work then on the characteristics and uses of points (they bound lines, mark crossings, anchor circles, and so on.) Nonetheless, we are always left with a certain discomfort, anxiety even, that we have not fully apprehended the essence of a point. In fact, in my case, the essence of a point, includes that which causes me discomfort. Whereas I can accept that the space in which the point resides is continuously covered by peanut butter, I can't swallow the notion of a point existing without extension. The fact, that the essence of a point is inaccessible coupled with the fact that its inaccessibility is part of its essence lead me to believe that it exists not in the realm of my mind but in another realm.

Now then, the common notions appear to exist in the realm of human thought and the point in another realm. Where then does geometry reside? The fact that the common notions are based on logic led us to conclude that the notions and therefore geometry resides in our minds. Yet, we must look further. What is it about logic that is so appealing? We say that the common notions are self-evident. The logic that if A=B & B=C, then A=C appears to be very different than the logic that if I'm hungry I should eat or if a baby cries it needs care. If something is self-evident **from what horizon** is it putting forth its *self-evidentness*.

Interestingly, the *self-evidentness* of the common notions doesn't provoke the enigmatic statements nor the anxiety associated with the definition of the point. There are at least three reasons that the common notions do not *appear* problematic. First, when we say that they are self-evident we automatically frame them or encapsulate them in a way that says that they do not require further examination. In fact, if we do not question from what horizon they are self-evident, then they are indeed self-evident. The term self-evident allows us to skip the hard work and if we want "keep on truckin" with the geometry and never look back. Secondly, each of the notions includes a concept on which we can focus – equality of equals in number 1, addition and subtraction of equals in 2 and 3, greater than and less than in number 5. By focusing on the notion at hand we are distracted again away from question of the horizon of *self-evidentness.* Finally, the common notions jive quite well with our daily experience. If the twins, Molly and Mary, both have three candy bars and mother takes away a Kit-Kat (my favorite) from each they will still be equal. Our daily experience with addition, subtraction, equality, coinciding, and greater than and less than all make the common notions, well, common. However, we should, must in fact, ask how is it possible that we are able to understand equality and the others. From where do we have an *a priori* ability to apprehend equality and do commerce with it? When we begin to ask this question the acceptance of the common notions becomes, like the point, very problematic.

Part III – Who am I to eat peanut butter?

A point ought to be equal to a point. With this assumption in place, and without further justification let us agree that the realm of the point and that from which the *self-evidentness* of the common-notions springs forth are the same. Let us now, investigate our relationship with this realm and the being that we call geometry.

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